MATHEMATICS FOR ECONOMICS SKILLS ASSESSMENT (MESA)
LEARNING GOALS
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Mathematics skills are essential to the study of economics. This document explicitly lists the math skills that students need in order to succeed in an introductory course in economics (e.g., Principles of Microeconomics or Principles of Macroeconomics). These skills, tested in MESA-Foundations, are included in section A (Arithmetic), B (Geometry and Graphs), and C (Algebra).

Section D (Calculus) contains the additional learning goals that are often required of students in intermediate level economics courses (e.g., Intermediate Microeconomics, Intermediate Macroeconomics, Economic Statistics, or Econometrics). Furthermore, there are extra learning goals for sections B (Geometry and Graphs) and C (Algebra) that are grouped under Intermediate Skills. These skills are tested in MESA-Intermediate.

This document also includes a list of "advanced" mathematical learning goals which relate to higher order calculus and static constrained optimization problems. Few undergraduate economics courses expect students to have these skills at the beginning of the term, but some courses teach them explicitly. The Mathematics for Economics Skills Assessment (MESA) intentionally does not include questions which test these learning goals.

FOUNDATION SKILLS

A. ARITHMETIC

1. Compute arithmetic expressions containing positive and negative numbers, sums, products, differences, quotients, exponents, and parenthetic expressions using correct order of operations.

2. Translate between proportions, ratios, fractions, decimals, and percentages.

3. Compute expressions involving addition, subtraction, multiplication, and division of proportions, ratios, fractions, decimals, and percentages.

4. Convert quantities measured in one unit to another.

5. Translate between unit changes, percentage changes, and percentage point changes.

6. Translate word problems (i.e., problems set up in English) into arithmetic expressions.

B. GEOMETRY & GRAPHS

1. Be able to represent algebraic relationships graphically
    a. Represent linear relationships graphically
    b. Represent non-linear relationships graphically

2. Read values off graphs of functions using appropriate units.

3. Read off and interpret growth rates from graphs of functions.

4. Be able to solve equations using graphs.

5. Describe behavior of functions using graphs.

6. Compute areas of rectangles, triangles, and trapezoids.

7. Interpret bar charts, pie charts, and histograms.

C. ALGEBRA

1. Understand how functions work.
    (a) Calculate and interpret the value of a function for numeric and arbitrary values of an exogenous variable.
    (b) Describe function behavior, given a linear or non-linear function
    (c) Calculate values of composite functions [e.g. $f(g(x))$]

2. Solve equations with simple polynomials, exponentials, and logs. Solutions may be numeric or contain parameters.

3. Solve systems of up to 3 linear equations with 3 unknowns.

4. Write down functional relationships.
   (a) Write down linear functions that describe given "data" (points, graphs, tables, etc.)

5. Translate word problems/descriptions to functional relationships

6. Know the definition of the inverse function and apply it to linear functions.

INTERMEDIATE SKILLS
B. GEOMETRY & GRAPHS

8. Be able to solve inequalities using graphs.

C. ALGEBRA

4. Write down functional relationships.
   (b) Substitute variables to obtain new formulations of the same function
   (c) Reparametrize functions

7. Obtain the inverses of non-linear functions.

D. CALCULUS

1. Compute limits of function; i.e., Describe the behavior of a function as its argument approaches some value.

2. Approximate the slope of the curve given tabular data

3. Take a derivative of a function of a single variable.
   (a) Take derivatives of polynomials, logs, exponentials.
   (b) Apply the product rule, the quotient rule, and the chain rule.

4. Take a first and second partial derivative of a function of multiple variables.

5. Describe the behavior of a function using derivatives
   (a) Find a maximand of a function (i.e., x such that $f'(x) = 0$ and $f''(x) < 0$)
   (b) Find critical points of the function (maximizers, minimizers, saddle points).
   (c) Substitute constraints into a maximization problem to transform it into an unconstrained maximization problem.

6. Integrate linear functions, polynomials, exponentials, and logs
   (a) Obtain an indefinite integral
   (b) Calculate a definite integral and interpret it as the area under the curve

"ADVANCED" CALCULUS LEARNING GOALS (Not Included in MESA)

These skills are often acquired as a part of a variety of Intermediate and Advanced Economics courses and are not treated and tested as pre-existing skills in the Mathematics for Economics Skills Assessment (MESA).

1. Solve constrained optimization problems using Lagrangians.
   (a) Set up the Lagrangian given the problem's objective function and constraints.
   (b) Solve for the critical points using the First Order Conditions
   (c) Check for the optima using the Second Order Conditions
   (d) Interpret the Lagrange multiplier given the context of the problem

2. Calculate double and triple integrals.
   (a) Interpret double integrals as "volume" under the surface.

3. Perform integration by parts.

4. Perform monotonic transformations of functions.